For the reactions, take moments about D:

$$9R_a = (122 \times 6) + (76 \times 3) + (12.6 \times 4.5) = 732 + 228 + 56.7 = 1016.7$$
$$R_a = 1016.7/9 = 112.97\,\text{kN}$$
$$R_d = (122 + 76 + 12.6) - 112.97 = 97.63\,\text{kN}$$

$$\text{Ultimate moment at B} = 112.97 \times 3 - \frac{12.6}{9} \times \frac{3^2}{2} = 332.61\,\text{kN m}$$

$$\text{Ultimate moment at C} = 97.63 \times 3 - \frac{12.6}{9} \times \frac{3^2}{2} = 286.59\,\text{kN m}$$

Since the beam is not fully restrained laterally, the buckling resistance moment M_b of the section needs to be checked in comparison with the applied equivalent uniform moment $\bar{M}$ to ensure that

$$\bar{M} = mM_A \leqslant M_b = p_b S_x$$

By reference to the bending moment diagram shown in Figure 5.25c, the critical unrestrained length will be BC where the maximum moment occurs.

The self-weight UDL is relatively insignificant and it is therefore satisfactory to consider the beam to be unloaded between restraints. Hence n is 1.0 and m is obtained from BS 5950 Table 18. We have

$$\beta = \frac{\text{smaller end moment}}{\text{larger end moment}} = \frac{M \text{ at C}}{M \text{ at B}} = \frac{286.59}{332.61} = 0.86$$

Therefore by interpolation from Table 18, $m = 0.93$. The maximum moment on length BC is $M_A = M$ at B = 332.61 kN m. Hence

$$\bar{M} = mM_A = 0.93 \times 332.61 = 309.33\,kN\,m$$

The effective length L_E of BC is 3.0 m.

By reference to Table 5.9, reproduced from the Steel Construction Institute design guide, a 457 × 191 × 74 kg/m UB has a buckling resistance moment of 355 kN m when n is 1 and L_E is 3.0 m. Therefore let us check this section in bending, shear and deflection.

The relevant properties for the section from tables are as follows:

Plastic modulus $S_x = 1660\,\text{cm}^3$

$D = 457.2\,\text{mm}$ $t = 9.1\,\text{mm}$ $T = 14.5\,\text{mm}$ $d = 407.9\,\text{mm}$

Section classification: plastic

Since $T = 14.5\,\text{mm} < 16\,\text{mm}$, $p_y = 275\,\text{N/mm}^2$.

Check the section for combined moment and shear as follows.

Maximum moment and coexistent shear at B

Ultimate shear at B is $F_v = 108.77\,\text{kN}$; $M = 332.61\,\text{kN m}$

Shear capacity of section is $P_v = 0.6 p_y t D$

$$= 0.6 \times 275 \times 9.1 \times 457.2 = 686\,486\,N$$
$$= 686\,\text{kN} > 108.77\,\text{kN}$$

This is satisfactory. Furthermore, $0.6P_v = 0.6 \times 686 = 412\,\text{kN}$. Therefore

$$F_v = 108.77\,\text{kN} < 0.6P_v = 412\,\text{kN}$$

Hence the shear load is low and the moment capacity is as follows:

$$M_{cx} = p_y S_x = 275 \times 1660 \times 10^3 = 456.5 \times 10^6\,\text{N mm} = 456.5\,\text{kN m} > 332.61\,\text{kN m}$$

Maximum shear and coexistent moment at A

Ultimate shear at A is $F_v = 112.97\,\text{kN}$; $M = 0$

P_v is again $686\,\text{kN} > 112.97\,\text{kN}$

Buckling resistance

The lateral torsional buckling resistance has already been satisfied by selecting a section from Table 5.9 with a buckling resistance moment M_b greater than the equivalent uniform moment $\bar{M}$. However, the method of calculating the buckling resistance moment in accordance with BS 5950 will be included here for reference.

The buckling resistance moment of the section is given by

$$M_b = p_b S_x$$

The bending strength p_b is obtained from Table 5.5 in relation to p_y and λ_{LT}. We have $p_y = 275\,\text{N/mm}^2$ and

$$\lambda_{LT} = nuv\lambda$$

Now $n = 1.0$, and $u = 0.876$ from section tables. Next $\lambda = L_E/r_y$, where $L_E = 1.0L$ in this instance from Table 5.6; L is the distance BC between restraints; and $r_y = 4.19\,\text{cm} = 4.19 \times 10\,\text{mm}$ from section tables. Thus

$$\lambda = \frac{L_E}{r_y} = \frac{1.0 \times 3000}{4.19 \times 10} = 71.6$$

Here $x = 33.9$ from section tables. Thus $\lambda/x = 71.6/33.9 = 2.11$, and so $v = 0.95$ by interpolation from Table 5.7.

Finally, therefore,

$$\lambda_{LT} = nuv\lambda = 1.0 \times 0.876 \times 0.95 \times 71.6 = 59.6$$

Using the values of p_y and λ_{LT}, $p_b = 214\,\text{N/mm}^2$ from Table 5.5. In conclusion,

$$M_b = p_b S_x = 214 \times 1660 \times 10^3 = 355.2 \times 10^6\,\text{N mm} = 355.2\,\text{kN m} > 309.33\,\text{kN m}$$

Thus $\bar{M} < M_b$, and therefore the lateral torsional buckling resistance of the section is adequate.

Deflection

Since the loading on this beam is not symmetrical, the calculations needed to determine the actual deflection are quite complex. A simpler approach is to calculate the deflection due to an equivalent UDL and compare it with the permitted limit of span/360. If this proves that the section is adequate then there would be no need to resort to more exact calculations.